GENERALIZED MODEL OF THE MECHANICS OF THIN-WALLED STRUCTURES MADE OF COMPOSITE MATERIALS

É. I. Grigolyuk and G. M. Kulikov

UDC 624.074.001:678.067

Independent kinematic and static hypotheses were used in [1] to construct a variant of the theory of elastic multilaminate anisotropic shells in which the order of the resolvent equations depends on the number of layers. This makes it possible to study local effects in shells made of composite materials. When the broken line hypothesis is employed, the tangential components of the displacement vector and stress and strain tensors are distributed linearly through the thickness of each layer. Here, we examine a variant of the theory of multilayered anisotropic shells in which the tangential displacements are distributed nonlinearly through the thickness of the layers in accordance with a generalized broken line hypothesis [2]. This approach is of interest in calculations of the stress—strain state in the immediate vicinity of the ends of a composite shell.

1. Let a shell of constant thickness h be composed of N elastic anisotropic layers. Each layer is of the thickness h_k. The inside surface of the shell is referred to curvilinear orthogonal coordinates α_i that are reckoned along the lines of principal curvature. We reckon the transverse coordinate z in the direction of an increase in the external normal to the reference surface. We use the equation $z = \delta_\ell$ ($\ell = 1, 2, \ldots, N-1$) to specify the interface between adjacent layers of the shell. We also introduce the standard notation: k_i represents the curvatures of the coordinate lines; A_i represents the Lamé parameters; q is the normal load; δ_{kn} is the Kronecker symbol. Here and below, $i, j = 1, 2$; $k, n = 1, 2, \ldots, N$.

We will use the approximation from [1] for the transverse shear stresses $\sigma_{i3}^{(k)}$. We will assume that they are distributed through the thickness of the k-th layer in accordance with the law

$$\sigma_{i3}^{(k)}=f_0(z)\mu_i^{(0)}+f_k(z)\mu_i^{(k)}, \tag{1.1}$$

where $f_0(z)$ and $f_k(z)$ are functions which are continuous in the region [0, h] and which satisfy the conditions ($\delta_0 = 0$, $\delta_N = h$)

$$f_0(0)=f_0(h)=0;\quad f_k(z)=0;\quad z\in[0,\delta_{k-1}]\cup[\delta_k,h]. \tag{1.2}$$

No limitations are placed on the form of the functions $f_0(z)$ and $f_k(z)$, and only in solving specific problems do we assume that they are quadratic parabolas. It is specific from Eq. (1.1) and (1.2) that the stresses $\sigma_{i3}^{(k)}$ are continuous functions of the coordinate z in the region [0, h], including on the interfaces $z = \delta_\ell$.

For the components of the displacement vector $u_i^{(k)}$, $u_3^{(k)}$, we adopt an independent approximation [2]

$$u_i^{(k)}=u_i+z\theta_i+\sum_{n=1}^{N}\pi_{kn}\beta_i^{(n)}+g(z)\beta_i^{(k)};\quad u_3^{(k)}=w. \tag{1.3}$$

Here, u_i and w are the tangential and normal displacements of points of the reference plane; π_{kn} are elements of a square N × N matrix. The sum of the elements of the k-th line is equal to zero ($\xi = 0$):

Moscow Institute of Automotive Mechanics. Tambov Institute of Chemical Engineering. Translated from Mekhanika Kompozitnykh Materialov, No. 4, pp. 698-704, July-August, 1988. Original article submitted November 2, 1987.

0191-5665/88/2404-0537$12.50

$$\|\pi_{kn}\| = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ \zeta_1-\zeta_0 & -\zeta_1 & 0 & \dots & 0 & 0 \\ \zeta_1-\zeta_0 & \zeta_2-\zeta_1 & -\zeta_2 & \dots & 0 & 0 \\ \dots \\ \zeta_1-\zeta_0 & \zeta_2-\zeta_1 & \zeta_3-\zeta_2 & \dots & \zeta_{N-1}-\zeta_{N-2} & -\zeta_{N-1} \end{bmatrix}.$$

We determine the remaining quantities from the formulas

$$\theta_i = k_i u_i - \frac{1}{A_i}\frac{\partial w}{\partial \alpha_i}; \quad g(z) = \int_0^z f_0(y)\,dy; \ \zeta_m = g(\delta_m) \quad (m=0, 1, \dots, N). \tag{1.4}$$

Kinematic hypothesis (1.3) as sufficiently general in character, since it naturally leads to the familiar hypotheses of shell theory. Thus, having set $g(z) = z$ in (1.3) and considering the above-noted property of the matrix $||\pi_{kn}||$, we arrive at the broken line hypothesis [3]. Here, for the functions $\alpha_i^{(k)}$ introduced in [3], we have the formula $\alpha_i^{(k)} = h_k(\beta_i^{(k)} + \theta_i)$. If we take $\beta_i^{(k)} = \beta_i$ and again use the property of the matrix $||\pi_{kn}||$, we obtain a kinematic hypothesis which is often used in refined shell theories – in particular, in [4, 5]. The order of the resolvent equations in this case is obviously independent of the number of layers. Finally, assuming that $\beta_i^{(k)} = 0$, we arrive at the Kirchhoff–Love kinematic hypothesis [6].

2. Let us present the strain relations of the problem. We insert the displacements from (1.3) into equations in [7] which determine the strain tensor in the case of the simplest nonlinear variant of shell theory, using a quadratic approximation. Then ignoring the terms $k_i z$ compared to unity, we obtain the formulas

$$\varepsilon_{ij}^{(k)} = E_{ij} + zK_{ij} + \sum_{n=1}^{N} \pi_{kn} R_{ij}^{(n)} + g(z) R_{ij}^{(k)}; \quad \varepsilon_{i3}^{(k)} = f_0(z)\beta_i^{(k)}; \quad \varepsilon_{33}^{(k)} = 0. \tag{2.1}$$

In (2.1), we used the notation

$$E_{11} = \varepsilon_1 + 1/2\theta_1^2; \ E_{12} = \omega_1 + \omega_2 + \theta_1\theta_2; \ K_{11} = \varkappa_1; \ K_{12} = \tau_1 + k_1\omega_2 + \tau_2 + k_2\omega_1;$$

$$\begin{gathered} \varepsilon_1 = \frac{1}{A_1}\frac{\partial u_1}{\partial \alpha_1} + \frac{1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2} u_2 + k_1 w; \ \omega_1 = \frac{1}{A_1}\frac{\partial u_2}{\partial \alpha_1} - \frac{1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2} u_1; \ \varkappa_1 = \frac{1}{A_1} \times \\ \times \frac{\partial \theta_1}{\partial \alpha_1} + \frac{1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2}\theta_2; \ \ \tau_1 = \frac{1}{A_1}\frac{\partial \theta_2}{\partial \alpha_1} - \frac{1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2}\theta_1; \ \ R_{11}^{(k)} = \frac{1}{A_1}\frac{\partial \beta_1^{(k)}}{\partial \alpha_1} + \\ + \frac{1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2}\beta_2^{(k)}; \ \ R_{12}^{(k)} = \frac{A_2}{A_1}\frac{\partial (A_2^{-1}\beta_2^{(k)})}{\partial \alpha_1} + \frac{A_1}{A_2}\frac{\partial (A_1^{-1}\beta_1^{(k)})}{\partial \alpha_2} \ (1 \rightleftarrows 2). \end{gathered} \tag{2.2}$$

Equations (2.1) and (2.2) permit an asymptotic solution to the corresponding relations of a refined shell theory of the Timoshenko type [5]. For this, we should set $\beta_i^{(k)} = \beta_i$. Additionally assuming $\beta_i = 0$ and omitting the nonlinear terms in (2.2), we arrive at the classical strain relations in [6].

It should be noted that the presence of the load terms in Eqs. (2.1) seriously complicates the entire subsequent analysis, but it also makes it possible to describe the nonlinear dependence of the strain-tensor components on the transverse coordinate z.

3. We will use the mixed variational principle to obtain the equations of equilibrium of multilayered anisotropic shells and the corresponding boundary conditions. This approach naturally allows for reduction of the three-dimensional problem of elasticity theory to a two-dimensional problem of shell theory while simultaneously resolving the contradictions inherent in the original system of independent static and kinematic hypotheses (1.1), (1.3). The approach was discussed quite thoroughly in [1, 5, 8] and thus will not be explained in detail here. Without showing their derivation, we will present the equations of equilibrium in the unit forces and moments

$$\begin{gathered} \frac{\partial (A_2T_1)}{\partial \alpha_1} - \frac{\partial A_2}{\partial \alpha_1}T_2 + \frac{1}{A_1}\frac{\partial (A_1^2 S)}{\partial \alpha_2} + \frac{\partial (k_1A_1H)}{\partial \alpha_2} + k_2\frac{\partial A_1}{\partial \alpha_2}H + \\ + k_1A_1A_2N_1 = 0; \ \ \frac{\partial (A_2N_1)}{\partial \alpha_1} + \frac{\partial (A_1N_2)}{\partial \alpha_2} - A_1A_2(k_1T_1 + k_2T_2) = -A_1A_2 q; \end{gathered} \tag{3.1}$$

$$\frac{\partial(A_2\Phi_1^{(k)})}{\partial\alpha_1}-\frac{\partial A_2}{\partial\alpha_1}\Phi_2^{(k)}+\frac{1}{A_1}\frac{\partial(A_1^2\Psi^{(k)})}{\partial\alpha_2}-A_1A_2Q_1^{(k)}=0;\ N_1=\frac{1}{A_1A_2}\times$$
$$\times\left[\frac{\partial(A_2M_1)}{\partial\alpha_1}-\frac{\partial A_2}{\partial\alpha_1}M_2+\frac{1}{A_1}\frac{\partial(A_1^2H)}{\partial\alpha_2}-A_1A_2(T_1\theta_1+S\theta_2)\right]\quad(1\rightleftarrows 2),$$

along with integral elastic relations for the transverse components of the stress and strain tensors

$$\sum_{k=1}^{N}\int_{\delta_{k-1}}^{\delta_k}(\varepsilon_{13}^{(k)}-a_{45}^{(k)}\sigma_{23}^{(k)}-a_{55}^{(k)}\sigma_{13}^{(k)})f_0(z)\,dz=0;$$
$$\int_{\delta_{k-1}}^{\delta_k}(\varepsilon_{13}^{(k)}-a_{45}^{(k)}\sigma_{23}^{(k)}-a_{55}^{(k)}\sigma_{13}^{(k)})f_k(z)\,dz=0\quad(1\rightleftarrows 2;\ 4\rightleftarrows 5),\tag{3.2}$$

where $a_{lm}^{(k)}$ ($l, m=4, 5$) are the transverse shear compliances of the k-th layer. For the tangential components of the stress and strain tensors, the relations of the generalized Hooke's law are satisfied exactly:

$$\sigma_{11}^{(k)}=b_{11}^{(k)}\varepsilon_{11}^{(k)}+b_{12}^{(k)}\varepsilon_{22}^{(k)}+b_{16}^{(k)}\varepsilon_{12}^{(k)};\quad \sigma_{22}^{(k)}=b_{12}^{(k)}\varepsilon_{11}^{(k)}+b_{22}^{(k)}\varepsilon_{22}^{(k)}+$$
$$+b_{26}^{(k)}\varepsilon_{12}^{(k)};\quad \sigma_{12}^{(k)}=b_{16}^{(k)}\varepsilon_{11}^{(k)}+b_{26}^{(k)}\varepsilon_{22}^{(k)}+b_{66}^{(k)}\varepsilon_{12}^{(k)}.\tag{3.3}$$

Let us return to the equilibrium equations of the shell. We will express the unit forces and moments used in (3.1) as follows

$$T_1=\sum_{k=1}^{N}T_1^{(k)};\ S=\sum_{k=1}^{N}S^{(k)};\ M_1=\sum_{k=1}^{N}M_1^{(k)};\ H=\sum_{k=1}^{N}H^{(k)};\ T_1^{(k)}=\int_{\delta_{k-1}}^{\delta_k}\sigma_{11}^{(k)}dz;$$
$$S^{(k)}=\int_{\delta_{k-1}}^{\delta_k}\sigma_{12}^{(k)}dz;\quad M_1^{(k)}=\int_{\delta_{k-1}}^{\delta_k}\sigma_{11}^{(k)}z\,dz;\ H^{(k)}=\int_{\delta_{k-1}}^{\delta_k}\sigma_{12}^{(k)}z\,dz;$$
$$Q_1^{(k)}=\int_{\delta_{k-1}}^{\delta_k}\sigma_{13}^{(k)}f_0(z)\,dz;\quad \Phi_1^{(k)}=\int_{\delta_{k-1}}^{\delta_k}\sigma_{11}^{(k)}g(z)\,dz+\sum_{n=1}^{N}\pi_{nk}T_1^{(n)};$$
$$\Psi^{(k)}=\int_{\delta_{k-1}}^{\delta_k}\sigma_{12}^{(k)}g(z)\,dz+\sum_{n=1}^{N}\pi_{nk}S^{(n)};\quad T_{12}^{(k)}=S^{(k)}+k_2H^{(k)}\quad(1\rightleftarrows 2).\tag{3.4}$$

Having inserted the tangential stresses from (3.3) into Eqs. (3.4) and having integrated with allowance for the strain relations (2.1), we arrive at other relations which link the unit forces and moments with the kinematic characteristics of the shell. We will write these new relations in matrix form:

$$\begin{bmatrix}\mathbf{T}\\ \mathbf{M}\\ \Phi^{(1)}\\ \vdots\\ \Phi^{(N)}\end{bmatrix}=\begin{bmatrix}A & B & U^{(1)} & \dots & U^{(N)}\\ B & C & V^{(1)} & \dots & V^{(N)}\\ U^{(1)} & V^{(1)} & W^{(11)} & \dots & W^{(1N)}\\ \vdots & \vdots & \vdots & & \vdots\\ U^{(N)} & V^{(N)} & W^{(N1)} & \dots & W^{(NN)}\end{bmatrix}\begin{bmatrix}\mathbf{E}\\ \mathbf{K}\\ \mathbf{R}^{(1)}\\ \vdots\\ \mathbf{R}^{(N)}\end{bmatrix};$$
$$\mathbf{T}=[T_1,T_2,S]^T;\quad \mathbf{M}=[M_1,M_2,H]^T;\quad \Phi^{(k)}=[\Phi_1^{(k)},\Phi_2^{(k)},\Psi^{(k)}]^T;$$
$$\mathbf{E}=[E_{11},E_{22},E_{12}]^T;\quad \mathbf{K}=[K_{11},K_{22},K_{12}]^T;\quad \mathbf{R}^{(k)}=[R_{11}^{(k)},R_{22}^{(k)},R_{12}^{(k)}]^T,\tag{3.5}$$

where A, B, C, $U^{(k)}$, $V^{(k)}$, $W^{(kn)}$ are square 3 × 3 stiffness matrices of the multilayered anisotropic shell. The elements of the matrix can be expressed through the tangential stiffnesses of the layers $b_{lm}^{(k)}$ ($l, m=1, 2, 6$) by means of simple but rather cumbersome formulas. The latter are therefore not presented here.

We insert the transverse components of the stress and strain tensors from (1.1), (1.2) into Eqs. (3.2), (3.4) and, considering the notation

$$\lambda_k=\int\limits_{\delta_{k-1}}^{\delta_k} f_0^2(z)\,dz;\quad \lambda_{kr}=\int\limits_{\delta_{k-1}}^{\delta_k} f_k(z)f_r(z)\,dz\quad (r=0,k);$$
$$\chi_k=\lambda_k-\frac{\lambda_{k0}^2}{\lambda_{kk}};\quad \tau_{lm}=\sum_{k=1}^{N}\chi_k a_{lm}^{(k)}\quad (l,m=4,5); \tag{3.6}$$
$$q_{lm}^{(kn)}=\frac{\chi_k\chi_n\tau_{lm}}{\tau_{44}\tau_{55}-\tau_{45}^2}+\delta_{kn}\frac{\lambda_{n0}^2}{\lambda_{nn}}\frac{a_{lm}^{(n)}}{a_{44}^{(n)}a_{55}^{(n)}-(a_{45}^{(n)})^2}\quad (l,m=4,5),$$

we arrive at formulas for the generalized transverse forces of the k-th layer

$$Q_1^{(k)}=\sum_{n=1}^{N}\left(q_{44}^{(kn)}\beta_1^{(n)}-q_{45}^{(kn)}\beta_2^{(n)}\right)\quad (1\rightleftarrows 2;\ 4\rightleftarrows 5). \tag{3.7}$$

Here, $q_{lm}^{(kn)}$ are the generalized transverse-shear coefficients. Equations (3.6) and (3.7) permit an asymptotic solution to the corresponding relations of the refined Timoshenko shell theory in [5]. Taking $\beta_i^{(k)}=\beta_i$ in (3.7) and summing $Q_i^{(k)}$ over k from 1 to N, we obtain the formula for the transverse-shear coefficients $q_{\ell m}$ from [5]:

$$q_{lm}=\sum_{k=1}^{N}\sum_{n=1}^{N}q_{lm}^{(kn)}\quad (l,m=4,5).$$

It should be noted that integral elastic relations (3.2) played an important role in the derivation of Eqs. (3.7). They are a link between the independent static and kinematic hypotheses (1.1) and (1.3), since they allow the "excess" functions $\mu_i(0)$ and $\mu_i^{(k)}$ to be expressed in terms of the independent functions $\beta_i^{(k)}$.

4. We will examine the axisymmetric deformation of a multilayered anisotropic shell of revolution. In this case, all of the quantities characterizing the stress–strain state of the shell will be functions of only a single variable α_1. We introduce the 4N + 8 solution vector:

$$\begin{aligned}\mathbf{Y}=[&T_1, N_1, M_1, \Phi_1^{(1)},\dots,\Phi_1^{(N)}, S+2k_2H, \Psi^{(1)},\dots,\Psi^{(N)},\\ &u_1, w, \theta_1, \beta_1^{(1)},\dots,\beta_1^{(N)}, u_2, \beta_2^{(1)},\dots,\beta_2^{(N)}]^T.\end{aligned} \tag{4.1}$$

In accordance with Eq. (4.1), this makes it possible to represent the resolvent equations of the problem in the form of a normal system of ordinary differential equations of the order 4N + 8. We write this system in matrix form:

$$\frac{1}{A_1}\frac{d\mathbf{Y}}{d\alpha_1}=\mathbf{F}(\alpha_1,\mathbf{Y}). \tag{4.2}$$

We have the following formulas for the components of the vector of the right sides F

$$\begin{aligned}&F_1=\rho(Y_1-T_2)-k_1Y_2;\quad F_2=k_1Y_1+\rho Y_2+k_2T_2-q;\quad F_3=\rho(Y_3-M_2)+Y_2+\\ &+Y_1Y_{2N+7}+S\theta_2;\quad F_{3+k}=\rho(Y_{3+k}-\Phi_2^{(k)})+Q_1^{(k)};\quad F_{N+4}=2\rho Y_{N+4}+k_2(T_2\theta_2+\\ &+SY_{2N+7});\quad F_{N+4+k}=2\rho Y_{N+4+k}+Q_2^{(k)};\quad F_{2N+5}=E_{11}-k_1Y_{2N+6}-1/2Y_{2N+7}^2;\\ &F_{2N+6}=k_1Y_{2N+5}-Y_{2N+7};\quad F_{2N+7}=K_{11};\quad F_{2N+7+k}=R_{11}^{(k)};\\ &F_{3N+8}=E_{12}-\rho Y_{3N+8}-Y_{2N+7}\theta_2;\quad F_{3N+8+k}=R_{12}^{(k)}-\rho Y_{3N+8+k}.\end{aligned} \tag{4.3}$$

Here, $\rho=-\dfrac{1}{A_1A_2}\dfrac{dA_2}{d\alpha_1}$. The first 2N + 4 equations of system (4.2), (4.3) follow from the equilibrium equations (3.1), while the other group of 2N + 4 equations follows directly from strain relations (2.2) and Eqs. (1.4).

The force characteristics of the shell T_2, S, M_2, $\Phi_2^{(k)}$, $Q_1^{(k)}$, $Q_2^{(k)}$ and its kinematic characteristics θ_2, E_{11}, E_{12}, K_{11}, $R_{11}^{(k)}$, $R_{12}^{(k)}$, entering into the right sides of system (4.2)-(4.3),

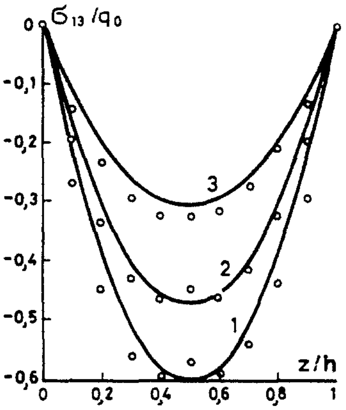

Fig. 1

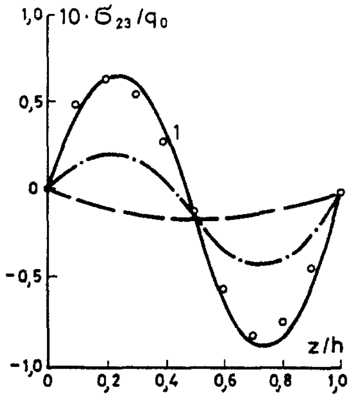

Fig. 2

must be expressed through the components of the vector Y. For this, we use Eqs. (3.5), (3.7), (1.4), and (2.2).

We augment normal system of equations (4.2) with 2N + 4 boundary conditions on each end of the closed shell of revolution:

$$Y_m(\alpha^*_1)\,l_m + Y_{2N+4+m}(\alpha^*_1)\,(1-l_m)=0;$$
$$Y_m(\alpha^{**}_1)\,l_{2N+4+m} + Y_{2N+4+m}(\alpha^{**}_1)\,(1-l_{2N+4+m})=0. \tag{4.4}$$

In (4.4), the characteristics of the boundary conditions l_m, l_{2N+4+m} $(m=1,\ 2,\ldots,2N+4)$ take values of 0, 1 and establish any combination of homogeneous kinematic and static boundary conditions on the ends of the shell $\alpha_1=\alpha^*_1$, $\alpha_1=\alpha^{**}_1$.

In concluding this section, we note that nonlinear boundary-value problem (4.2), (4.4) can be reduced to a sequence of linear boundary-value problems and can be realized on a computer in accordance with algorithms developed to numerically solve nonlinear problems of the statics of multilayered anisotropic shells of revolution [1, 5, 8]. Realization of the given algorithm will of course have special features connected with the high order of the system of resolvent equations, but in concept the algorithm differs little from those used in [1, 5, 8]. The algorithm to solve nonlinear boundary-value problem (4.2), (4.4) was realized in the form of a set of standard procedures written in the programming language PL/1(0). All of the numerical calculations were performed on an ES-1060 computer.

5. As a numerical example, we will examine a reinforced two-layer cylindrical shell loaded by a normal load $q=-q_0\sin^8(\pi\alpha_1/l)$, where ℓ is the length of the shell. The layers of the shell are arranged antisymmetrically, with the angles of lay of the fibers $\gamma_k=(-1)^{k-1}\gamma$ $(k=1,2)$. The geometric parameters of the shell were taken from [9]: $l=2$; $h=0{,}1$; $R_1=1$, where R_1 is the distance from the axis of rotation to the inside surface of the shell. The initial characteristics of an elementary reinforced layer of 0.05 thickness were also given in [9]. The boundary conditions on the ends of the shell $\alpha_1 = 0$ and $\alpha_1 = \ell$ are assigned by means of the characteristics of the boundary conditions $\ell_m = 0$ and $l_{8+m}=0$ $(m=1,2,\ldots,8)$.

The solid lines in Figs. 1 and 2 show the distribution of the transverse shear stresses σ_{i3} through the thickness of the packet in the shell section with the coordinate $\alpha_1 = 0.8$. For comparison, the circles show the results obtained in [9], where the shell was examined from the viewpoint of the linear theory of elasticity. In connection with this, the curves in Figs. 1 and 2 were obtained by integration of linear boundary-value problem (4.2), (4.4). The diagrams of the transverse shear stresses σ_{13} were constructed with $\gamma = \pi/6$, $\pi/4$, $\pi/3$ (curves 1-3 in Fig. 1, respectively), while the diagrams of σ_{23} were constructed with $\gamma = \pi/6$ (curve 1 in Fig. 2). The broken curve in Fig. 2 corresponds to calculations performed on the basis of a Timoshenko-type shell theory [8], while the dot–dash curve corresponds to a refined theory of the Timoshenko type [5] based on static hypothesis (1.1). However, the order of the normal system of equations in this theory is equal to 12 and does not depend on the number of layers.

Let us use geometrically nonlinear theory to analyze the distribution of the tangential and transverse shear stresses in a fixed, cross-reinforced cylindrical shell. We realize the

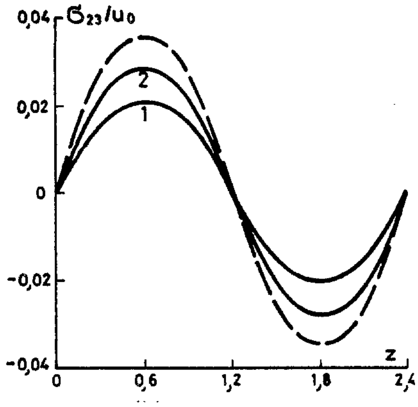

Fig. 3

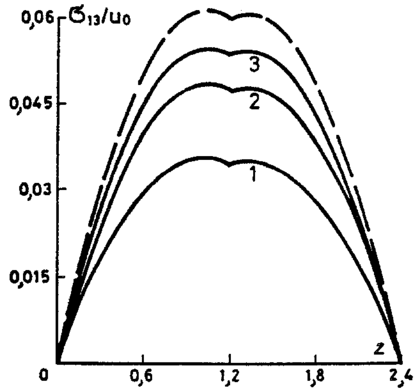

Fig. 4

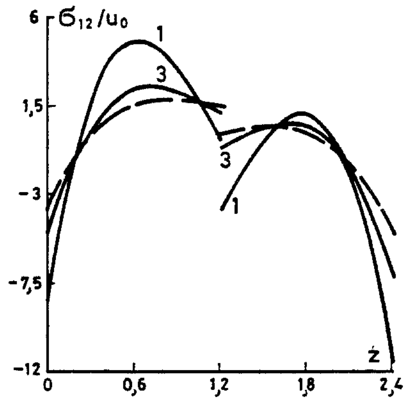

Fig. 5

problem numerically for a shell tensioned in the axial direction and having the geometric parameters $h = 2.4$ mm, $\ell = 100$ mm, $R_1 = 50$ mm. The shell is made of a rubber-cord composite. The mechanical characteristics of the composite were presented in [1].

We will assume that the ends of the shell are displaced an identical distance u_0. Figures 3 and 4 show the distribution of the transverse shear stresses through the thickness of the packet in a cross section of the two-layer shell located 10 mm from the right end. The angle of reinforcement $\gamma = 30°$. Figure 5 shows the distribution of the tangential shear stresses through the thickness of the packet on the right end of the shell. The solid lines show results of solution of the problem using the geometrically nonlinear theory of shells with $u_0 = 2$ mm, and $u_0 = 1$ mm, and $u_0 = 0.5$ mm (curves 1-3). The dashed curves were obtained on the basis of the linear theory of shells (any values of u_0). It can be seen that allowing for geometric nonlinearity has a significant influence on the effect of anisotropy in a cross-reinforced rubber-cord shell. It should also be noted that the tangential stresses are distributed through the thickness of the layers in accordance with a law differing from the linear law in [1].

CONCLUSIONS

1. A kinematic hypothesis which generalizes the broken line hypothesis was used to construct a variant of the theory of multilayered anisotropic shells with allowance for local effects, making it possible to study the actual law of distribution of components of the stress and strain tensors through the thickness of the layers.

2. The combined effect of anisotropy and geometric nonlinearity on the stress state of cross-reinforced shells was studied. It was found that failure to allow for anisotropy leads to serious errors in the design of composite shells with a small number of layers.

3. It was established that in the short edge-effect region near the ends of the shell, the tangential forces are distributed through the thickness of the layers in accordance with a law which is nonlinear.

LITERATURE CITED

1. É. I. Grigolyuk and G. M. Kulikov, "Theory and numerical solution of problems of the statics of multilayered reinforced shells," Mekh. Kompozitn. Mater., No. 4, 643-650 (1986).
2. É. I. Grigolyuk and G. M. Kulikov, "Stress concentration near the free end of a cross-reinforced shell," Transactions of the XIV All-Union Conference on the Theory of Plates and Shells, Kutaisi (1987), pp. 398-403.
3. É. I. Grigolyuk and P. P. Chulkov, "Nonlinear equations of thin elastic solid anisotropic shallow shells with a stiff filler," Izv. Akad. Nauk SSSR, Mekh., No. 5, 68-80 (1965).
4. A. G. Teregulov, "Toward a theory of multilayered anisotropic shells," Issled. Teor. Plastin. Oboloohek. Nos. 6, 7, 762-767 (1970).
5. É. I. Grigolyuk and G. M. Kulikov, "Design of radial tires on the basis of a generalized Timoshenko theory," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 4, 166-174 (1984).
6. V. V. Novozhilov, Theory of Thin Shells [in Russian], Leningrad (1962).
7. V. V. Novozhilov, Principles of the Nonlinear Theory of Elasticity [in Russian], Leningrad (1948).
8. É. I. Grigolyuk and G. M. Kulikov, "Numerical solution of problems of the statics of geometrically nonlinear anisotropic multilayered shells of revolution," Mekh. Kompozitn. Mater., No. 3, 443-452 (1981).
9. A. T. Vasilenko and N. D. Pankratova, "Solution of the problem of the stress state of anisotropic nonuniform cylinders," Prikl. Mekh., 20, No. 8, 11-18 (1984).