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On The First-Order Seven-Parameter Plate Theory

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Abstract: A first-order seven-parameter theory of plates undergoing finite rotations is developed. The precise representation of large rigid-body motions in the displacement patterns of the plate elements is considered. The fundamental unknowns consist of six in-plane and transverse displacements of the face planes and an additional transverse displacement of the midplane. Because of thickness stretching the 3D equations of Hooke's law are utilized. However, no thickness locking can be observed in the proposed plate model. This is demonstrated by analytical and numerical studies of the isotropic plate bending.

1. Introduction

One of the main requirements of the modern plate theory that is intended for the general finite element formulation is that it must lead to strain-free modes for arbitrary rigid-body motions [1, 2]. The adequate representation of rigid-body motions is a necessary condition if the element is to have the good accuracy and convergence properties. Therefore, when an inconsistent non-linear plate theory is used to construct any finite element, erroneous straining modes under arbitrarily large rigid-body motions may be appeared. This problem has been studied for the finite rotation Timoshenko beam [3], Mindlin plate [4] and Timoshenko-Mindlin-type shell [5, 6] theories. Such sort of theories may be treated as the first-order four-parameter beam and six-parameter plate/shell models and, therefore, so-called thickness locking [7, 8] can occur.

Herein, the more general study on the basis of the finite rotation first-order seven-parameter plate theory taking into account the transverse normal deformation response is considered. As unknowns six in-plane and transverse displacements of the face planes of the plate and an additional transverse displacement of the midplane are chosen. Such choice of displacements gives the possibility to deduce non-linear strain-displacement relationships, which are objective, i.e., invariant under all rigid-body motions. It should be mentioned that in some works (see e.g. [9]) developing the solid-shell concept, displacement vectors of the face and middle surfaces are also utilized. But in our first-order plate theory selecting as unknowns the displacements of the face and middle planes has a principally another mechanical sense and allows one to deduce non-linear strain-displacement relationships with aforementioned attractive properties.

As has been already said the six-parameter plate model on the basis of the complete 3D constitutive equations is deficient because thickness locking occurs. This is due to the fact that the adopted linear displacement field in the thickness direction results in a constant transverse normal strain, which in turn causes artificial stiffening of the plate element in the case of non-vanishing Poisson's ratios. To prevent thickness locking at the finite element level the enhanced assumed strain method [7] can be applied. In order to circumvent a locking phenomenon at the mechanical level and computational one as well, the 3D constitutive equations have to be modified [6, 10]. However, the use of complete 3D constitutive laws within the plate analysis is of great importance for engineering applications. Thus, the first-order seven-parameter plate model is best suited for this purpose because such a model is optimal with respect to a number of degrees of freedom employed.

2. Strain-displacement relationships

Let us consider a plate of the uniform thickness h. The plate may be defined as a 3D body bounded by two planes S^- and S^+, located at the distances d^- and d^+ measured with respect to the reference plane S, and the edge boundary cylindrical surface Ω that is perpendicular to the reference plane. Let the reference plane S be referred to the Cartesian coordinate system x_1 and x_2. The x_3 axis is oriented along the normal direction. The initial and current configurations of the plate are shown in Fig. 1.

The position vectors of the arbitrary point in the plate body and points belonging the face and middle planes can be expressed as

$$\mathbf{R} = N^-\mathbf{R}^- + N^+\mathbf{R}^+, \tag{1}$$

$$\mathbf{R}^\mathrm{I} = x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + d^\mathrm{I}\mathbf{e}_3 \quad (\mathrm{I} = -, \mathrm{M}, +), \tag{2}$$

$$N^- = \frac{1}{h}\left(d^+ - x_3\right), \quad N^+ = \frac{1}{h}\left(x_3 - d^-\right), \quad d^\mathrm{M} = \frac{1}{2}\left(d^- + d^+\right), \tag{3}$$

where $N^\pm(x_3)$ are the linear through-thickness shape functions of the plate; d^M is the distance from the reference plane to the middle one.

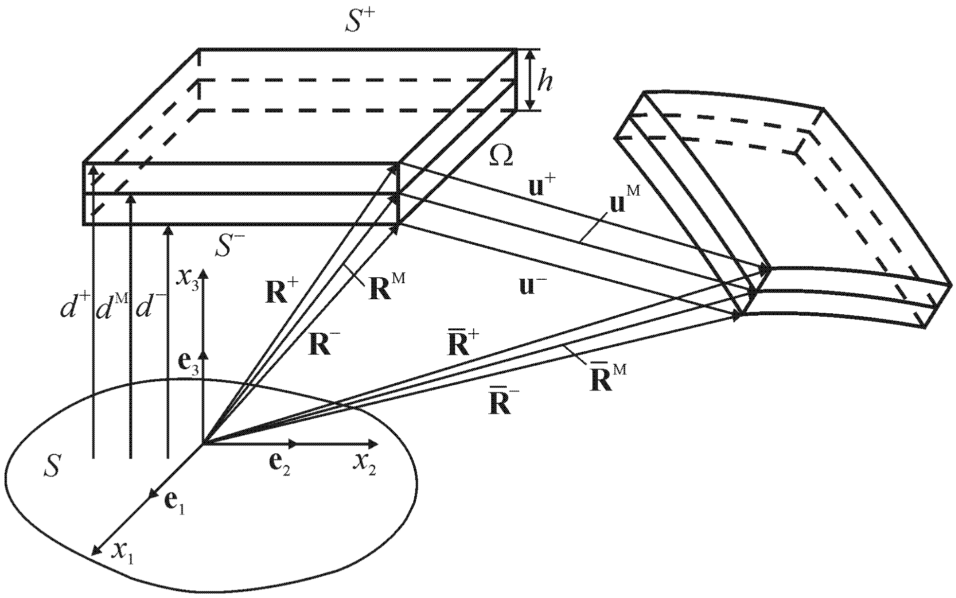

Fig. 1. Initial and current configurations of the plate

The position vectors of points in the plate in its current configuration are given by

$$\bar{\mathbf{R}} = L^{-}\bar{\mathbf{R}}^{-} + L^{\mathrm{M}}\bar{\mathbf{R}}^{\mathrm{M}} + L^{+}\bar{\mathbf{R}}^{+}, \tag{4}$$

$$\bar{\mathbf{R}}^{\mathrm{I}} = \mathbf{R}^{\mathrm{I}} + \mathbf{u}^{\mathrm{I}} \qquad (\mathrm{I} = -, \mathrm{M}, +), \tag{5}$$

$$L^{-} = N^{-}\left(N^{-} - N^{+}\right), \qquad L^{\mathrm{M}} = 4N^{-}N^{+}, \qquad L^{+} = N^{+}\left(N^{+} - N^{-}\right), \tag{6}$$

where $L^{\pm}(x_3)$ and $L^{\mathrm{M}}(x_3)$ are the quadratic through-thickness shape functions of the plate; $\mathbf{u}^{\pm}(x_1, x_2)$ and $\mathbf{u}^{\mathrm{M}}(x_1, x_2)$ are the displacement vectors of the face and middle planes defined as

$$\mathbf{u}^{\mathrm{I}} = \sum_i u_i^{\mathrm{I}} \mathbf{e}_i \qquad (\mathrm{I} = -, \mathrm{M}, +). \tag{7}$$

The components of the Green-Lagrange strain tensor can be written as

$$2\varepsilon_{ij} = \bar{\mathbf{R}}_{,i}\bar{\mathbf{R}}_{,j} - \mathbf{R}_{,i}\mathbf{R}_{,j}, \tag{8}$$

where the abbreviation $(\ )_{,i}$ implies the partial derivatives with respect to coordinates x_i and indices i, j take the values 1, 2 and 3. In the following developments Greek indices α, β running from 1 to 2 are also utilized.

Substituting derivatives of position vectors (1) and (4) into the 3D strain-displacement relationships (8) and assuming that quadratic and higher-order terms in the thickness direction are negligible, one obtains

$$2\varepsilon_{ij} = N^{-}\varepsilon_{ij}^{-} + N^{+}\varepsilon_{ij}^{+}, \tag{9}$$

where ε_{ij}^{-} and ε_{ij}^{+} are the strains of the bottom and top planes expressed as

$$2\varepsilon_{\alpha\beta}^{\pm} = \mathbf{u}_{,\alpha}^{\pm}\mathbf{e}_{\beta} + \mathbf{u}_{,\beta}^{\pm}\mathbf{e}_{\alpha} + \mathbf{u}_{,\alpha}^{\pm}\mathbf{u}_{,\beta}^{\pm}, \tag{10}$$

$$2\varepsilon_{\alpha 3}^{\pm} = \left(\boldsymbol{\beta} \pm \frac{4}{h}\mathbf{w}\right)\mathbf{e}_{\alpha} + \mathbf{u}_{,\alpha}^{\pm}\left(\mathbf{e}_3 + \boldsymbol{\beta} \pm \frac{4}{h}\mathbf{w}\right),$$

$$2\varepsilon_{33}^{\pm} = \left(\boldsymbol{\beta} \pm \frac{4}{h}\mathbf{w}\right)\left(2\mathbf{e}_3 + \boldsymbol{\beta} \pm \frac{4}{h}\mathbf{w}\right),$$

where

$$\boldsymbol{\beta} = \frac{1}{h}\left(\mathbf{u}^{+} - \mathbf{u}^{-}\right), \qquad \mathbf{w} = \bar{\mathbf{u}} - \mathbf{u}^{\mathrm{M}}, \qquad \bar{\mathbf{u}} = \frac{1}{2}\left(\mathbf{u}^{-} + \mathbf{u}^{+}\right). \tag{11}$$

It is seen that instead of the midplane displacement vector $\mathbf{u}^{\mathrm{M}}$ the convenient difference displacement vector $\mathbf{w}$ is involved into a set of unknown functions. Thus, a set of fundamental unknowns consists of displacement vectors $\mathbf{u}^{-}$, $\mathbf{u}^{+}$ and $\mathbf{w}$.

Remark 1. It can be verified that approximate and exact components of the Green-Lagrange strain tensor satisfy the following linking conditions:

$$\varepsilon_{ij}\left(d^{\pm}\right) = \varepsilon_{ij}^{\mathrm{exact}}\left(d^{\pm}\right) = \varepsilon_{ij}^{\pm}.$$

As pointed out previously the exact components $\varepsilon_{ij}^{\text{exact}}$ depend on the quadratic and higher-order terms in the thickness direction. This remark is illustrated by means of Fig. 2.

Proposition 1. The Green-Lagrange strains (9) are invariant under large rigid-body motions.

Proof. An arbitrarily large rigid-body displacement can be defined by

$$(\mathbf{u})^{\text{Rigid}} = \mathbf{\Delta} + (\mathbf{\Phi} - \mathbf{I})\mathbf{R}, \tag{12}$$

where $\mathbf{\Delta}$ is the constant displacement (translation) vector; $\mathbf{I}$ is the identity matrix; $\mathbf{\Phi}$ is the orthogonal rotation matrix. In particular, rigid-body displacements of the face and middle planes are

$$\left(\mathbf{u}^{\text{I}}\right)^{\text{Rigid}} = \mathbf{\Delta} + (\mathbf{\Phi} - \mathbf{I})\mathbf{R}^{\text{I}} \qquad (\text{I} = -,\ \text{M},\ +). \tag{13}$$

Allowing for Eqs (11) and (13), one can verify that

$$(\bar{\mathbf{u}})^{\text{Rigid}} = \frac{1}{2}\left(\mathbf{u}^{-} + \mathbf{u}^{+}\right)^{\text{Rigid}} = \mathbf{\Delta} + (\mathbf{\Phi} - \mathbf{I})\mathbf{R}^{\text{M}} = \left(\mathbf{u}^{\text{M}}\right)^{\text{Rigid}}.$$

Thus,

$$(\mathbf{w})^{\text{Rigid}} = \left(\bar{\mathbf{u}} - \mathbf{u}^{\text{M}}\right)^{\text{Rigid}} = \mathbf{0} \tag{14}$$

and

$$(\boldsymbol{\beta})^{\text{Rigid}} = \mathbf{\Phi}\mathbf{e}_3 - \mathbf{e}_3, \qquad \left(\mathbf{u}_{,\alpha}^{\pm}\right)^{\text{Rigid}} = \mathbf{\Phi}\mathbf{e}_\alpha - \mathbf{e}_\alpha. \tag{15}$$

It can be shown by using Eqs (14) and (15) that strains (10) are all zero in a general large rigid-body motion

$$2\left(\varepsilon_{ij}^{\pm}\right)^{\text{Rigid}} = \left(\mathbf{\Phi}\mathbf{e}_i\right)\left(\mathbf{\Phi}\mathbf{e}_j\right) - \mathbf{e}_i\mathbf{e}_j = 0. \tag{16}$$

This conclusion is true because an orthogonal transformation retains the scalar product of vectors. So, due to Eq. (16) the Green-Lagrange strains (9) exactly represent arbitrarily large rigid-body motions. □

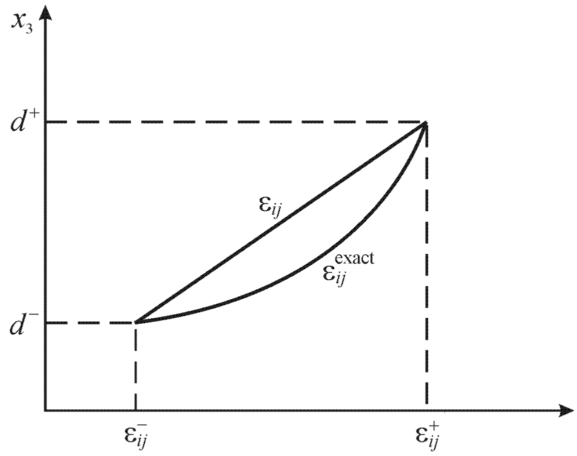

Fig. 2. Distribution of approximate and exact components of the Green-Lagrange strain tensor through the thickness

Further we introduce the basis assumption for the proposed plate model. The in-plane displacements are considered to be linear in the thickness direction, whereas the transverse displacement is parabolic through the thickness of the plate [11], that is,

$$\mathbf{w} = w_3 \mathbf{e}_3. \tag{17}$$

Substituting displacements (7) in strain-displacement relationships (10) and taking into account the adopted assumption (17), we derive the following strain-displacement relationships of the first-order seven-parameter plate model:

$$2\varepsilon^{\pm}_{\alpha\beta} = u^{\pm}_{\alpha,\beta} + u^{\pm}_{\beta,\alpha} + u^{\pm}_{1,\alpha}u^{\pm}_{1,\beta} + u^{\pm}_{2,\alpha}u^{\pm}_{2,\beta} + u^{\pm}_{3,\alpha}u^{\pm}_{3,\beta}, \tag{18}$$

$$2\varepsilon^{\pm}_{\alpha 3} = \beta_\alpha + u^{\pm}_{3,\alpha} + \beta_1 u^{\pm}_{1,\alpha} + \beta_2 u^{\pm}_{2,\alpha} + u^{\pm}_{3,\alpha}\left(\beta_3 \pm \frac{4}{h} w_3\right),$$

$$\varepsilon^{\pm}_{33} = \beta_3 \pm \frac{4}{h} w_3 + \frac{1}{2}\left(\beta_1^2 + \beta_2^2\right) + \frac{1}{2}\left(\beta_3 \pm \frac{4}{h} w_3\right)^2,$$

where

$$\beta_i = \frac{1}{h}\left(u_i^+ - u_i^-\right). \tag{19}$$

As we shall see later, strains (9) in conjunction with Eqs (18) and (19) provide a very simple and convenient way to overcome thickness locking in the case of utilizing the complete 3D constitutive equations because only seven displacements u_i^-, u_i^+ and w_3 are introduced.

3. Governing equations of plates

The equilibrium equations of the plate can be obtained by applying the principle of the virtual work. Herein, for simplicity we restrict ourselves to the geometrically linear effects. As a result, one derives seven equilibrium equations

$$T_{1i,1} + T_{2i,2} = p_i^- - p_i^+, \tag{20}$$

$$M_{1i,1} + M_{2i,2} - \frac{2}{h} T_{i3} = -p_i^- - p_i^+, \tag{21}$$

$$M_{33} = 0, \tag{22}$$

where p_i^- and p_i^+ are the tractions acting on the bottom and top planes in x_i directions; T_{ij} and M_{ij} are the stress resultants defined as

$$T_{ij} = H^-_{ij} + H^+_{ij}, \qquad M_{ij} = H^+_{ij} - H^-_{ij}, \tag{23}$$

$$H^{\pm}_{ij} = \int_{d^-}^{d^+} \sigma_{ij} N^{\pm} dx_3.$$

The natural boundary conditions for the rectangular plate at edges $x_1 = 0$ and $x_1 = a$ will be

$$\left(H^{\pm}_{1i} - H^{*\pm}_{1i}\right)\delta u^{\pm}_i = 0, \qquad H^{*\pm}_{1i} = \int_{d^-}^{d^+} q_i N^{\pm} dx_3, \tag{24}$$

$$H_{13}^{*}\delta w_3 = 0, \qquad H_{13}^{*} = \int_{d^-}^{d^+} q_3 L^{\mathrm{M}} dx_3, \tag{25}$$

where q_i are the tractions acting on these edges in x_i directions. The boundary conditions at edges $x_2 = 0$ and $x_2 = b$ can be written in a similar way.

It seems surprising that Eqs (22) and (25) must be fulfilled regardless of loading and boundary conditions. This is due to the fact that all derivatives of the difference displacement w_3 have been omitted in strain-displacement relationships (18), in order, first, to arrive at their simplest form and, second, to utilize the complete 3D constitutive equations. However, Eq. (22) can be easily satisfied in the case of the uniform distribution of transverse normal stresses in the thickness direction. Really, substituting $\sigma_{33} = \sigma_0$ in Eq. (23) and integrating, one finds

$$H_{33}^{-} = H_{33}^{+} = \frac{1}{2}\sigma_0 h$$

and, therefore, the equilibrium equation (22) is satisfied exactly.

It is interesting to note that in some works ([6, 10], among others), which develop six-parameter shell models, the transverse normal stress is also assumed to be constant in the thickness direction. The constant σ_{33} assumption is one of the best thickness locking remedies in a six-parameter shell formulation but leads to the modification of 3D constitutive equations. It should be mentioned that Eq. (22) is also discussed in [7], where has been developed an efficient seven-parameter shell formulation on the basis of the enhanced assumed strain concept. In this work it is said that in the finite element formulation the last equilibrium equation (22) is carried out in a weak sense.

In regards to the boundary condition (25), one can conclude that a displacement w_3 is not a subject to any kinematic constraints. As a result, the transverse normal strain is not vanishing through the thickness at clamped edges. The same problem arises in all seven-parameter shell models (see e.g. [7, 9, 12]). However, this contradiction is not principal because in the finite element formulation the difference displacement w_3 is condensed on the element level. In practice, this implies that instead of the midplane displacement u_3^{M} the average displacement $\overline{u}_3$ should be employed.

In isotropic elasticity the 3D constitutive equations are written as

$$\sigma_{ij} = \lambda\delta_{ij}\left(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}\right) + 2G\varepsilon_{ij}, \tag{26}$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \qquad G = \frac{E}{2(1+\nu)},$$

where E and G are Young's and shear moduli; ν is Poisson's ratio; λ is a Lamé parameter; δ_{ij} is Kronecker's delta.

Using Eqs (18), (23) and (26) in equilibrium equations (20) – (22), one derives the following differential equations in terms of generalized displacements $\overline{u}_i$, β_i and w_3:

$$2(1-\nu)\overline{u}_{1,11} + (1-2\nu)\overline{u}_{1,22} + \overline{u}_{2,12} + 2\nu\beta_{3,1} = 0, \tag{27}$$

$$2(1-\nu)\overline{u}_{2,22} + (1-2\nu)\overline{u}_{2,11} + \overline{u}_{1,12} + 2\nu\beta_{3,2} = 0, \tag{28}$$

$$2\beta_{1,11} + (1-\nu)\beta_{1,22} + (1+\nu)\beta_{2,12} - \frac{12(1-\nu)}{h^2}\left(\overline{u}_{3,1} + \beta_1\right) = 0, \tag{29}$$

$$2\beta_{2,22}+(1-\nu)\beta_{2,11}+(1+\nu)\beta_{1,12}-\frac{12(1-\nu)}{h^2}\left(\bar{u}_{3,2}+\beta_2\right)=0, \tag{30}$$

$$\Delta\bar{u}_3+\beta_{1,1}+\beta_{2,2}=\frac{1}{Gh}\left(p_3^- - p_3^+\right), \tag{31}$$

$$\Delta\beta_3-\frac{24\nu}{(1-2\nu)h^2}\left(\bar{u}_{1,1}+\bar{u}_{2,2}+\frac{1-\nu}{\nu}\beta_3\right)=-\frac{6}{Gh^2}\left(p_3^- + p_3^+\right), \tag{32}$$

$$w_3=\frac{\nu h^2}{8(1-\nu)}\left[\Delta\bar{u}_3+\frac{1}{Gh}\left(p_3^+ - p_3^-\right)\right], \tag{33}$$

where Δ is the Laplace operator and

$$\bar{u}_i=\frac{1}{2}\left(u_i^- + u_i^+\right), \qquad \beta_i=\frac{1}{h}\left(u_i^+ - u_i^-\right), \qquad w_3=\bar{u}_3-u_3^{\mathrm{M}}. \tag{34}$$

Remark 2. The difference displacement w_3 does not appear in equilibrium equations (27) – (32) because this one has been eliminated with the help of Eq. (33), which explicitly defines w_3. It is important to note that Eqs (27) – (32) coincide with corresponding equilibrium equations of the six-parameter plate model based on the equivalent constant σ_{33} assumption [13].

Invoking an approach [11], we introduce new functions χ and φ such that

$$\beta_1=\chi_{,1}+\varphi_{,2}, \qquad \beta_2=\chi_{,2}-\varphi_{,1}. \tag{35}$$

Using Eq. (35) in equilibrium equations (29)–(31), one obtains

$$D\Delta\Delta\bar{u}_3=\frac{h^2}{6(1-\nu)}\Delta\left(p_3^- - p_3^+\right)-p_3^- + p_3^+, \qquad D=\frac{Eh^3}{12\left(1-\nu^2\right)}, \tag{36}$$

$$\Delta\varphi=\frac{12}{h^2}\varphi, \tag{37}$$

$$\chi=-\frac{h^2}{6(1-\nu)}\Delta\bar{u}_3-\bar{u}_3+\frac{(1+\nu)h}{3(1-\nu)E}\left(p_3^- - p_3^+\right). \tag{38}$$

So, we have four governing differential equations (27), (28), (32) and (36) in terms of generalized displacements $\bar{u}_i$ and β_3. It should be mentioned that Eq. (37) describes the well-known Reissner's edge effect [14].

4. Numerical example

A simply supported rectangular isotropic plate, depicted in Fig. 3, is subjected to the sinusoidally distributed pressure load

$$p_3^+ = p_0\sin\frac{\pi x_1}{a}\sin\frac{\pi x_2}{b}, \qquad p_3^- = 0,$$

where a and b are two in-plane dimensions of the plate.

We will search an exact solution of the problem in the following form:

$$\bar{u}_1=\bar{u}_{10}\cos\frac{\pi x_1}{a}\sin\frac{\pi x_2}{b}, \qquad \bar{u}_2=\bar{u}_{20}\sin\frac{\pi x_1}{a}\cos\frac{\pi x_2}{b},$$

$$\left(u_3^-, \bar{u}_3, u_3^{\mathrm{M}}, u_3^+, \beta_3\right)=\left(u_{30}^-, \bar{u}_{30}, u_{30}^{\mathrm{M}}, u_{30}^+, \beta_{30}\right)\sin\frac{\pi x_1}{a}\sin\frac{\pi x_2}{b}.$$

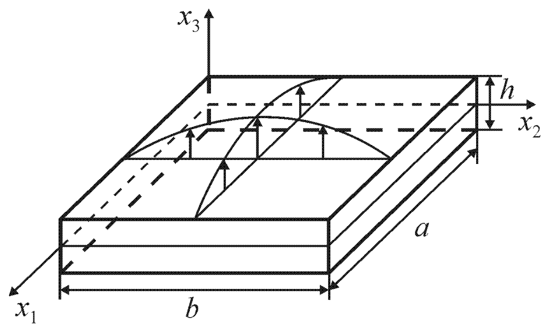

Fig. 3. Rectangular plate under sinusoidal loading:

$$a = b = 3;\ E = 10^7;\ \nu = 0{,}3;\ p_3^+ = p_0 \sin\frac{\pi x_1}{a}\sin\frac{\pi x_2}{b}$$

Table 1

Dimensionless transverse displacements in a center of the square plate

a/h	Displacement	Elasticity	FPT7	FPT6M	CPT	FPT6
3	U_3^-	3,882	4,002	4,002	2,803	3,487
	$\overline{U}_3$	4,161	4,266	4,266	2,803	3,751
	U_3^{M}	4,309	<u>4,595</u>	4,266	2,803	3,751
	U_3^+	4,440	4,530	4,530	2,803	4,015
10	U_3^-	2,912	2,932	2,932	2,803	2,417
	$\overline{U}_3$	2,915	2,934	2,934	2,803	2,420
	U_3^{M}	2,942	<u>2,964</u>	2,934	2,803	2,420
	U_3^+	2,917	2,937	2,937	2,803	2,422
30	U_3^-	2,815	2,817	2,817	2,803	2,302
	$\overline{U}_3$	2,815	2,817	2,817	2,803	2,302
	U_3^{M}	2,818	2,821	2,817	2,803	2,302
	U_3^+	2,815	2,817	2,817	2,803	2,303
100	U_3^-	2,804	2,804	2,804	2,803	2,289
	$\overline{U}_3$	2,804	2,804	2,804	2,803	2,289
	U_3^{M}	2,804	2,804	2,804	2,803	2,289
	U_3^+	2,804	2,804	2,804	2,803	2,289

After trivial calculations, one finds

$$\bar{u}_{30} = (1+\Theta)v_{30}, \qquad u_{30}^{\mathrm{M}} = [1+(1+0,75\nu)\Theta]v_{30},$$

$$\beta_{30} = \frac{6(1-\nu)^2 P}{4+(1-\nu)^2\Theta}, \qquad u_{30}^{\pm} = \bar{u}_{30} \pm \frac{1}{2}h\beta_{30},$$

$$v_{30} = \frac{Ph}{\Theta^2}, \qquad P = \frac{(1+\nu)p_0}{3(1-\nu)E}, \qquad \Theta = \frac{\pi^2\left(1+a^2/b^2\right)}{6(1-\nu)(a/h)^2},$$

where v_{30} is the transverse displacement in a center of the plate for the classical plate theory (**CPT**).

Table 1 lists dimensionless transverse displacements

$$\left(U_3^-, \bar{U}_3, U_3^{\mathrm{M}}, U_3^+\right) = \left(u_{30}^-, \bar{u}_{30}, u_{30}^{\mathrm{M}}, u_{30}^+\right)\frac{100Eh^3}{p_0a^4}$$

in a center of the square plate by using the present first-order plate theory (**FPT7**) for various values of the slenderness ratio a/h. A comparison with exact solutions of the elasticity theory [15], CPT [16] and the first-order six-parameter plate theory (**FPT6**) [13], based on the *full* constitutive equations, and the first-order six-parameter plate theory (**FPT6M**) [13], based on the constant σ_{33} assumption, is also given. It is seen that the FPT6 solution demonstrates significant thickness locking, whereas FPT7 and FPT6M ones perform well. Let us pay attention to equal values of the average displacement $\bar{U}_3$ for both FPT7 and FPT6M solutions, and excessive values of the midplane displacement U_3^{M} predicted by the FPT7 solution for thick plates (see underlined numbers in Tabl. 1). As has been pointed out already, this inconsistency is not principal for the proposed seven-parameter plate model, since for practical implementations instead of u_3^{M} the more appropriate average displacement $\bar{u}_3$ can be used.

5. Conclusions

The simple non-linear strain-displacement equations of the first-order seven-parameter plate model have been developed. These equations are attractive because they are objective, i.e., invariant under arbitrarily large rigid-body motions. Therefore, they may be used for the formulation of efficient plate elements undergoing finite rotations. However, the practical use of these equations requires the deep understanding of basis hypotheses underlying the proposed plate theory, in particular, the constant σ_{33} assumption. For this purpose, the geometrically linear bending of the isotropic plate is studied in detail.

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Ключевые слова и фразы: большие перемещения твердого тела; конечные повороты; теория пластин первого порядка.

Аннотация: Развита 7-параметричекая теория пластин первого порядка, подверженных конечным поворотам. Рассмотрено точное представление больших перемещений пластины как жесткого тела. В качестве искомых функций выбраны шесть тангенциальных и поперечных перемещений лицевых плоскостей и дополнительно поперечное перемещение срединной плоскости. Вследствие учета обжатия пластины по толщине использованы трехмерные уравнения закона Гука. Однако, предложенная модель пластины не подвержена запиранию по толщине. Это демонстрируется на примере изгиба изотропной пластины с использованием аналитических и численных методов.

Zur 7-parametrischen Theorie der Platten der ersten Ordnung

Zusammenfassung: Es ist die 7-parametrische Theorie der Platten der ersten Ordnung, die den endlichen Wendungen unterworfen sind, entwickelt. Es ist die genaue Vorstellung der grossen Umstellungen der Platte wie des harten Körpers untersucht. Als gesuchte Funktionen sind sechs Tangens- und Querumstellungen der Gesichtsebenen und die zusätzlich querlaufende Umstellung der Mittelebene gewählt. Infolge der Kontrolle der Plattenverformung nach der Dicke sind die dreidimensionalen Gleichungen des Hooke-Gesetzes verwendet. Doch, ist das angebotene Modell der Platte dem Sperren nach der Dicke nicht unterworfen. Es wird auf dem Beispiel der Biegung der Isotropplatte mit der Nutzung der analytischen und numerischen Methoden demonstriert.

Vers une théorie 7-paramétrique des plaques du premier ordre

Résumé: Est développée la théorie 7-paramétrique des plaques du premier ordre. Est examinée une représentation exacte sur les grands déplacements de la plaque comme un corps dur. En qualité des fonctions recherchées sont choisis six déplacements tangentiels et transversaux des plans de face et comme supplément – déplacement transversal du plan médian. Par suite du corroyage de la plaque par l'épaisseur sont utilisées les équations de trois mesures de Hooke. Néanmoins, le modèle proposé n'est pas soumis au blocage par l'épaisseur. Cela est montré à l'exemple du pliage de la plaque isotrope avec l'emploi des méthodes analitiques et numériques.
