

**THREE-DIMENSIONAL THERMOELASTIC ANALYSIS
OF LAMINATED ANISOTROPIC PLATES**

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Abstract: This paper presents the sampling surfaces (SaS) method applied to the three-dimensional (3D) analysis of steady-state problems of thermoelasticity for laminated anisotropic plates subjected to thermal loading. It is shown that the SaS method can be utilized efficiently for finding the 3D solutions for thermoelastic angle-ply composite plates in cylindrical bending with a specified accuracy by using the sufficient number of SaS located at Chebyshev polynomial nodes.

Introduction

Three-dimensional exact analysis of laminated anisotropic plates has attracted considerable attention over the past forty years. This is due to the fact that the validity of approximate plate theories and plate finite elements can be assessed by comparing their predictions with the 3D exact solutions. Pagano [1] presented the exact solution for the cylindrical bending of laminated composite plates with general layups. The response of a thermoelastic laminated anisotropic plate in cylindrical bending was investigated analytically in [2, 3]. The developments for antisymmetric angle-ply laminates in the framework of 3D thermoelasticity were carried out in article [4]. However, the reliable 3D exact solutions for thermoelastic laminated composite plates of general lay-up configurations can not be found in the current literature.

To solve such a problem, we invoke the efficient method of SaS developed recently in articles [5 – 8] for the analysis of laminated composite plates and shells. As SaS denoted here by $\Omega^{(n)1}, \Omega^{(n)2}, \dots, \Omega^{(n)I_n}$, we choose outer surfaces and any inner surfaces inside the n th layer of the plate and introduce temperatures $T^{(n)1}, T^{(n)2}, \dots, T^{(n)I_n}$ and displacement vectors $\mathbf{u}^{(n)1}, \mathbf{u}^{(n)2}, \dots, \mathbf{u}^{(n)I_n}$ of these surfaces as basic plate variables, where I_n is the total number of SaS chosen for each layer ($I_n \geq 3$). Such choice of temperatures and displacements with the consequent use of Lagrange polynomials of degree $I_n - 1$ in the thickness direction for each layer permits the representation of governing equations of the thermal laminated plate formulation in a very compact form. Note also that the origins of using the SaS can be found in contributions [9 – 14].

It should be mentioned that the developed approach with an arbitrary number of equally spaced SaS [5] does not work properly with the Lagrange polynomials of high

degree because the Runge's phenomenon can occur, which yields the wild oscillation at the edges of the interval when the user deals with any specific functions. If the number of equally spaced nodes is increased then the oscillations become even larger. However, the use of Chebyshev polynomial nodes [15] can help to improve significantly the behaviour of Lagrange polynomials of high degree for which the error will go to zero as $I_n \rightarrow \infty$. This fact gives an opportunity to derive the 3D exact solutions for thermal laminated anisotropic plates with a prescribed accuracy employing the sufficiently large number of SaS located at Chebyshev polynomial nodes.

Description of temperature field

Consider a laminated plate of the thickness h . Let the middle surface Ω be described by Cartesian coordinates x_1 and x_2 . The coordinate x_3 is oriented in the thickness direction. The transverse coordinates of SaS inside the n th layer are defined as

$$x_3^{(n)1} = x_3^{[n-1]}, \quad x_3^{(n)I_n} = x_3^{[n]},$$

$$x_3^{(n)m_n} = \frac{1}{2}(x_3^{[n-1]} + x_3^{[n]}) - \frac{1}{2}h_n \cos\left(\pi \frac{2m_n - 3}{2(I_n - 2)}\right), \quad (1)$$

where $x_3^{[n-1]}$ and $x_3^{[n]}$ are the transverse coordinates of layer interfaces $\Omega^{[n-1]}$ and $\Omega^{[n]}$ (Fig. 1); $h_n = x_3^{[n]} - x_3^{[n-1]}$ is the thickness of the n th layer; I_n is the number of SaS corresponding to the n th layer; the index n identifies the belonging of any quantity to the n th layer and runs from 1 to N ; N is the total number of layers; the index m_n identifies the belonging of any quantity to the inner SaS of the n th layer and runs from 2 to $I_n - 1$, whereas the indices i_n, j_n, k_n to be introduced later for describing all SaS of the n th layer run from 1 to I_n . Besides, the tensorial indices i, j, k, l range from 1 to 3 and Greek indices α, β range from 1 to 2.

It is worth noting that transverse coordinates of inner SaS (1) coincide with the coordinates of Chebyshev polynomial nodes [15]. This fact has a great meaning for a convergence of the SaS method [6 – 8].

The relation between the temperature T and the temperature gradient Γ is given by

$$\Gamma = \nabla T. \quad (2)$$

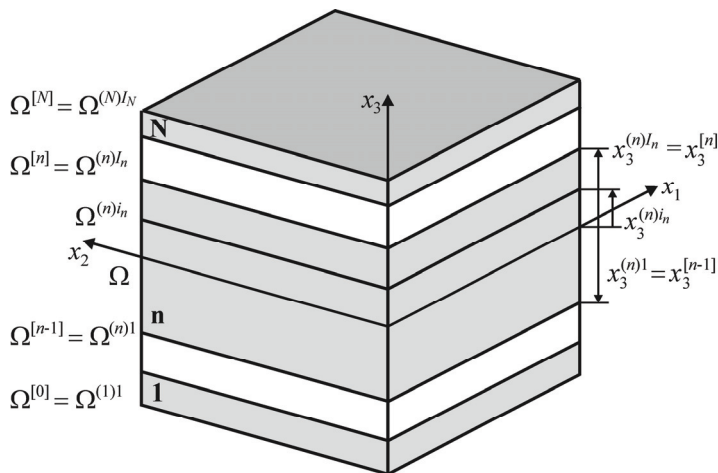


Fig. 1. Geometry of the laminated plate

In a component form it can be written as

$$\Gamma_i = T_{,i}, \quad (3)$$

where the symbol $(\dots)_{,i}$ stands for the partial derivatives with respect to coordinates x_i .

We start now with the first assumption of the proposed thermoelastic laminated plate formulation. Let us assume that the temperature and temperature gradient fields are distributed through the thickness of the n th layer as follows:

$$T^{(n)} = \sum_{i_n} L^{(n)i_n} T^{(n)i_n}; \quad x_3^{[n-1]} \leq x_3 \leq x_3^{[n]}, \quad (4)$$

$$\Gamma_i^{(n)} = \sum_{i_n} L^{(n)i_n} \Gamma_i^{(n)i_n}; \quad x_3^{[n-1]} \leq x_3 \leq x_3^{[n]}, \quad (5)$$

where $T^{(n)i_n}(x_1, x_2)$ are the temperatures of SaS of the n th layer $\Omega^{(n)i_n}$; $\Gamma_i^{(n)i_n}(x_1, x_2)$ are the components of the temperature gradient at the same SaS; $L^{(n)i_n}(x_3)$ are the Lagrange polynomials of degree $I_n - 1$ defined as:

$$T^{(n)i_n} = T(x_3^{(n)i_n}); \quad (6)$$

$$\Gamma_i^{(n)i_n} = \Gamma_i(x_3^{(n)i_n}); \quad (7)$$

$$L^{(n)i_n} = \prod_{j_n \neq i_n} \frac{x_3 - x_3^{(n)j_n}}{x_3^{(n)i_n} - x_3^{(n)j_n}}. \quad (8)$$

The use of relations (3), (4), (6) and (7) yields:

$$\Gamma_{\alpha}^{(n)i_n} = T_{,\alpha}^{(n)i_n}; \quad (9)$$

$$\Gamma_3^{(n)i_n} = \sum_{j_n} M^{(n)j_n}(x_3^{(n)i_n}) T^{(n)j_n}, \quad (10)$$

where $M^{(n)j_n} = L_{,3}^{(n)j_n}$ are the derivatives of Lagrange polynomials, which are calculated at SaS as follows:

$$M^{(n)j_n}(x_3^{(n)i_n}) = \frac{1}{x_3^{(n)j_n} - x_3^{(n)i_n}} \prod_{k_n \neq i_n, j_n} \frac{x_3^{(n)i_n} - x_3^{(n)k_n}}{x_3^{(n)j_n} - x_3^{(n)k_n}} \quad \text{for } j_n \neq i_n;$$

$$M^{(n)i_n}(x_3^{(n)i_n}) = - \sum_{j_n \neq i_n} M^{(n)j_n}(x_3^{(n)i_n}). \quad (11)$$

It is seen from (10) that the transverse component of the temperature gradient $\Gamma_3^{(n)i_n}$ is represented as a linear combination of temperatures of all SaS of the n th layer $T^{(n)j_n}$.

Description of displacement and strain fields

The strain tensor is given by

$$2\varepsilon_{ij} = u_{i,j} + u_{j,i}, \quad (12)$$

where u_i are the displacements of the plate. In particular, the strain tensor at SaS of the n th layer $\Omega^{(n)i_n}$ can be expressed as:

$$\begin{aligned}
2\varepsilon_{\alpha\beta}^{(n)i_n} &= 2\varepsilon_{\alpha\beta}(x_3^{(n)i_n}) = u_{\alpha,\beta}^{(n)i_n} + u_{\beta,\alpha}^{(n)i_n}; \\
2\varepsilon_{\alpha 3}^{(n)i_n} &= 2\varepsilon_{\alpha 3}(x_3^{(n)i_n}) = \beta_{\alpha}^{(n)i_n} + u_{3,\alpha}^{(n)i_n}; \\
\varepsilon_{33}^{(n)i_n} &= \varepsilon_{33}(x_3^{(n)i_n}) = \beta_3^{(n)i_n},
\end{aligned}
\tag{13}$$

where $u_i^{(n)i_n}(x_1, x_2)$ are the displacements of SaS; $\beta_i^{(n)i_n}(x_1, x_2)$ are the derivatives of displacements with respect to the thickness coordinate at SaS defined as:

$$u_i^{(n)i_n} = u_i(x_3^{(n)i_n}); \tag{14}$$

$$\beta_i^{(n)i_n} = u_{i,3}(x_3^{(n)i_n}). \tag{15}$$

The following step consists in a choice of the approximation of displacements and strains through the thickness of the n th layer. It is apparent that the displacement and strain distributions should be chosen similar to the thermal distributions (4) and (5). Thus, the second assumption of the developed thermoelastic laminated plate formulation can be written as:

$$u_i^{(n)} = \sum_{i_n} L^{(n)i_n} u_i^{(n)i_n}; \quad x_3^{[n-1]} \leq x_3 \leq x_3^{[n]}; \tag{16}$$

$$\varepsilon_{ij}^{(n)} = \sum_{i_n} L^{(n)i_n} \varepsilon_{ij}^{(n)i_n}; \quad x_3^{[n-1]} \leq x_3 \leq x_3^{[n]}. \tag{17}$$

The use of (15) and (16) yields

$$\beta_i^{(n)i_n} = \sum_{j_n} M^{(n)j_n}(x_3^{(n)i_n}) u_i^{(n)j_n}, \tag{18}$$

which is similar to (10). So, the key functions $\beta_i^{(n)i_n}$ of the proposed plate formulation are represented as a linear combination of displacements of SaS of the n th layer $u_i^{(n)j_n}$.

Variational formulation of heat conduction problem

The variational equation for the laminated plate can be written as

$$\delta J = 0, \tag{19}$$

where J is the basic functional of the heat conduction theory given by

$$J = \frac{1}{2} \iint_{\Omega} \sum_n \int_{x_3^{[n-1]}}^{x_3^{[n]}} q_i^{(n)} \Gamma_i^{(n)} dx_1 dx_2 dx_3 - \iint_{\bar{\Omega}} T Q_n d\Omega, \tag{20}$$

where $q_i^{(n)}$ are the components of the heat flux vector of the n th layer; Q_n is the specified heat flux on the boundary surface $\bar{\Omega} = \Omega^{[0]} + \Omega^{[N]} + \Sigma$, where Σ is the edge boundary surface of the plate.

Substituting the distribution (5) in functional (20) and introducing the heat flux resultants

$$R_i^{(n)i_n} = \int_{x_3^{[n-1]}}^{x_3^{[n]}} q_i^{(n)} L^{(n)i_n} dx_3, \tag{21}$$

one obtains

$$J = \frac{1}{2} \iint_{\Omega} \sum_n \sum_{i_n} R_i^{(n)i_n} \Gamma_i^{(n)i_n} dx_1 dx_2 - \iint_{\bar{\Omega}} T Q_n d\Omega. \quad (22)$$

Now, we accept the third assumption of the proposed thermal laminated plate formulation. Let the constitutive equations be the Fourier's heat conduction equations:

$$q_i^{(n)} = -k_{ij}^{(n)} \Gamma_j^{(n)}; \quad x_3^{[n-1]} \leq x_3 \leq x_3^{[n]}, \quad (23)$$

where $k_{ij}^{(n)}$ are the components of the thermal conductivity tensor of the n th layer.

Inserting the constitutive equations (23) in (21) and taking into account the distribution (5), we have

$$R_i^{(n)i_n} = - \sum_{j_n} \Lambda^{(n)i_n j_n} k_{ij}^{(n)} \Gamma_j^{(n)j_n}, \quad (24)$$

where

$$\Lambda^{(n)i_n j_n} = \int_{x_3^{[n-1]}}^{x_3^{[n]}} L^{(n)i_n} L^{(n)j_n} dx_3. \quad (25)$$

Variational formulation of thermomechanical problem

The variational equation for the thermoelastic laminated plate in the case of conservative loading can be written as

$$\delta \Pi = 0, \quad (26)$$

where Π is the basic functional of the theory of thermoelasticity given by:

$$\Pi = \frac{1}{2} \iint_{\Omega} \sum_n \int_{x_3^{[n-1]}}^{x_3^{[n]}} \left(\sigma_{ij}^{(n)} \varepsilon_{ij}^{(n)} - \eta^{(n)} \Theta^{(n)} \right) dx_1 dx_2 dx_3 - W; \quad (27)$$

$$W = \iint_{\Omega} \left(p_i^+ u_i^{[N]} - p_i^- u_i^{[0]} \right) dx_1 dx_2 + W_{\Sigma}, \quad (28)$$

where $\sigma_{ij}^{(n)}$ are the components of the stress tensor of the n th layer; $\eta^{(n)}$ is the entropy density of the n th layer; $u_i^{[0]} = u_i^{(1)1}$ and $u_i^{[N]} = u_i^{(N)I_N}$ are the displacements of bottom and top surfaces $\Omega^{[0]}$ and $\Omega^{[N]}$; p_i^- and p_i^+ are the loads acting on bottom and top surfaces; W_{Σ} is the work done by external loads applied to the edge surface Σ ; $\Theta^{(n)}$ is the temperature rise from the initial reference temperature T_0 defined as

$$\Theta^{(n)} = T^{(n)} - T_0. \quad (29)$$

Substituting the strain distribution (17) and the temperature distribution:

$$\Theta^{(n)} = \sum_{i_n} L^{(n)i_n} \Theta^{(n)i_n}; \quad x_3^{[n-1]} \leq x_3 \leq x_3^{[n]}, \quad (30)$$

which follows directly from relations (4) and (29) in the functional (27) and introducing the stress resultants

$$H_{ij}^{(n)i_n} = \int_{x_3^{[n-1]}}^{x_3^{[n]}} \sigma_{ij}^{(n)} L^{(n)i_n} dx_3 \quad (31)$$

and the entropy resultants

$$S^{(n)i_n} = \int_{x_3^{[n-1]}}^{x_3^{[n]}} \eta^{(n)} L^{(n)i_n} dx_3, \quad (32)$$

one obtains

$$\Pi = \frac{1}{2} \iint_{\Omega} \sum_n \sum_{i_n} \left(H_{ij}^{(n)i_n} \varepsilon_{ij}^{(n)i_n} - S^{(n)i_n} \Theta^{(n)i_n} \right) dx_1 dx_2 - W. \quad (33)$$

Finally, we introduce the fourth assumption of the proposed thermal laminated plate formulation. Let us consider the case of linear thermoelastic materials. Therefore, the constitutive equations [16] are written as follows:

$$\sigma_{ij}^{(n)} = C_{ijkl}^{(n)} \varepsilon_{kl}^{(n)} - \gamma_{ij}^{(n)} \Theta^{(n)}; \quad x_3^{[n-1]} \leq x_3 \leq x_3^{[n]}, \quad (34)$$

$$\eta^{(n)} = \gamma_{ij}^{(n)} \varepsilon_{ij}^{(n)} + \chi^{(n)} \Theta^{(n)}; \quad x_3^{[n-1]} \leq x_3 \leq x_3^{[n]}, \quad (35)$$

where $C_{ijkl}^{(n)}$ are the elastic constants of the n th layer; $\gamma_{ij}^{(n)}$ are the thermal stress coefficients of the n th layer; $\chi^{(n)}$ is the entropy-temperature coefficient defined as

$$\chi^{(n)} = \rho^{(n)} c_v^{(n)} / T_0, \quad (36)$$

where $\rho^{(n)}$ and $c_v^{(n)}$ are the mass density and the specific heat per unit mass of the n th layer at constant strain.

Inserting the constitutive equations (34) and (35) correspondingly in (31) and (32) and taking into consideration the strain and temperature field distributions (17) and (30), we have:

$$H_{ij}^{(n)i_n} = \sum_{j_n} \Lambda^{(n)i_n j_n} \left(C_{ijkl}^{(n)} \varepsilon_{kl}^{(n)j_n} - \gamma_{ij}^{(n)} \Theta^{(n)j_n} \right); \quad (37)$$

$$S^{(n)i_n} = \sum_{j_n} \Lambda^{(n)i_n j_n} \left(\gamma_{ij}^{(n)} \varepsilon_{ij}^{(n)j_n} + \chi^{(n)} \Theta^{(n)j_n} \right). \quad (38)$$

3D exact solution for laminated anisotropic plates in cylindrical bending

In this section, we study a laminated anisotropic plate in cylindrical bending subjected to temperature loading. The boundary conditions for the simply supported plate whose edges are maintained at the reference temperature are written as

$$\sigma_{11}^{(n)} = \sigma_{12}^{(n)} = u_3^{(n)} = \Theta^{(n)} = 0 \quad \text{at } x_1 = 0 \text{ and } x_1 = a, \quad (39)$$

where a is the width of the plate. To satisfy boundary conditions, we search the analytical solution of the problem by a method of Fourier series expansion:

$$\Theta^{(n)i_n} = \sum_r \Theta_r^{(n)i_n} \sin \frac{r\pi x_1}{a}; \quad (40)$$

$$u_{\alpha}^{(n)i_n} = \sum_r u_{\alpha r}^{(n)i_n} \cos \frac{r\pi x_1}{a}, \quad u_3^{(n)i_n} = \sum_r u_{3r}^{(n)i_n} \sin \frac{r\pi x_1}{a}, \quad (41)$$

where r is the wave number along the x_1 -direction. The external loads are also expanded in Fourier series.

Substituting the Fourier series (40) in (29) and using relations (9), (10), (22) and (24), one obtains

$$J = \sum_r J_r(\Theta_r^{(n)i_n}). \quad (42)$$

Invoking the variational equation (19), we arrive at the system of linear algebraic equations

$$\frac{\partial J_r}{\partial \Theta_r^{(n)i_n}} = 0 \quad (43)$$

of order K , where $K = \sum_n I_n - N + 1$. Thus, the temperatures $\Theta_r^{(n)i_n}$ of SaS of the n th layer can be easily found by using a method of Gaussian elimination.

Inserting (40) and (41) and Fourier series corresponding to mechanical loading in (13), (18), (28), (33), (37) and (38), we have

$$\Pi = \sum_r \Pi_r(u_{ir}^{(n)i_n}, \Theta_r^{(n)i_n}). \quad (44)$$

The use of relations (26) and (44) leads to a system of linear algebraic equations

$$\frac{\partial \Pi_r}{\partial u_{ir}^{(n)i_n}} = 0 \quad (45)$$

of order $3K$. The linear system (45) is solved through a method of Gaussian elimination.

The described algorithm was performed with the Symbolic Math Toolbox, which incorporates symbolic computations into the numeric environment of MATLAB. This in turn gives the possibility to derive the exact solutions of thermoelasticity for laminated anisotropic plates in cylindrical bending with a specified accuracy.

As a numerical example, we consider a two-layer angle-ply plate with the stacking sequence $[45/-45]$ and ply thicknesses $h_1 = h_2 = h/2$ made of the graphite-epoxy composite. The mechanical properties of the composite are taken as follows: $E_L = E_0$; $E_T = E_0/10$; $G_{LT} = E_0/20$; $G_{TT} = E_0/50$; $\nu_{LT} = \nu_{TT} = 0.25$; $\alpha_L = \alpha_0$; $\alpha_T = 7.2\alpha_0$; $k_L = 100k_0$; $k_T = k_0$; $\rho = 1800 \text{ Kg/m}^3$ and $c_v = 900 \text{ J/KgK}$, where $E_0 = 2 \times 10^{11} \text{ Pa}$; $\alpha_0 = 5 \times 10^{-6} \text{ 1/K}$ and $k_0 = 0.5 \text{ W/mK}$.

The plate is loaded on the top surface by the sinusoidally distributed temperature, while the bottom surface is maintained at the reference temperature, that is

$$\Theta^{[N]} = \Theta_0 \sin \frac{\pi x_1}{a}, \quad \Theta^{[0]} = 0, \quad (46)$$

where $\Theta_0 = 1 \text{ K}$ and $T_0 = 293 \text{ K}$. To compare the results derived with an exact solution [3], we accept $a = 1 \text{ m}$ and introduce dimensionless variables:

Results for a thick angle-ply plate

I_n	$\bar{u}_3(0)$	$\bar{\sigma}_{11}(-0,5)$	$\bar{\sigma}_{13}(-0,25)$	$\bar{\sigma}_{23}(0,25)$	$\bar{\sigma}_{33}(0,25)$	$\bar{\Theta}(0,25)$	$\bar{q}_3(0,25)$	$\bar{\eta}(0,25)$
3	1.7049	-2.1688	0.61235	-1.1563	7.8971	0.32457	-8.8888	364.07
5	1.6575	-2.0507	0.98221	-1.4364	0.05346	0.32719	-7.2800	366.22
7	1.6575	-2.0492	0.98142	-1.4367	0.15040	0.32714	-7.3220	366.19
9	1.6575	-2.0492	0.98141	-1.4366	0.14830	0.32714	-7.3216	366.19
11	1.6575	-2.0492	0.98141	-1.4366	0.14831	0.32714	-7.3215	366.19
13	1.6575	-2.0492	0.98141	-1.4366	0.14831	0.32714	-7.3215	366.19
Vel	1.6570	-2.0490	0.98000	-1.4400	0.14800	-	-	-

$$\begin{aligned} \bar{u}_3 &= u_3(a/2, z)/\alpha\alpha_0\Theta_0; \quad \bar{\sigma}_{11} = 10\sigma_{11}(a/2, z)/E_0\alpha_0\Theta_0; \quad \bar{\sigma}_{13} = 100\sigma_{13}(a/4, z)/E_0\alpha_0\Theta_0; \\ \bar{\sigma}_{23} &= 100\sigma_{23}(a/4, z)/E_0\alpha_0\Theta_0; \quad \bar{\sigma}_{33} = 100\sigma_{33}(a/2, z)/E_0\alpha_0\Theta_0; \quad \bar{\Theta} = \Theta(a/2, z)/\Theta_0; \\ \bar{q}_3 &= aq_3(a/2, z)/k_0\Theta_0; \quad \bar{\eta} = \eta(a/2, z)/E_0\alpha_0^2\Theta_0, \quad z = x_3/h. \end{aligned} \quad (47)$$

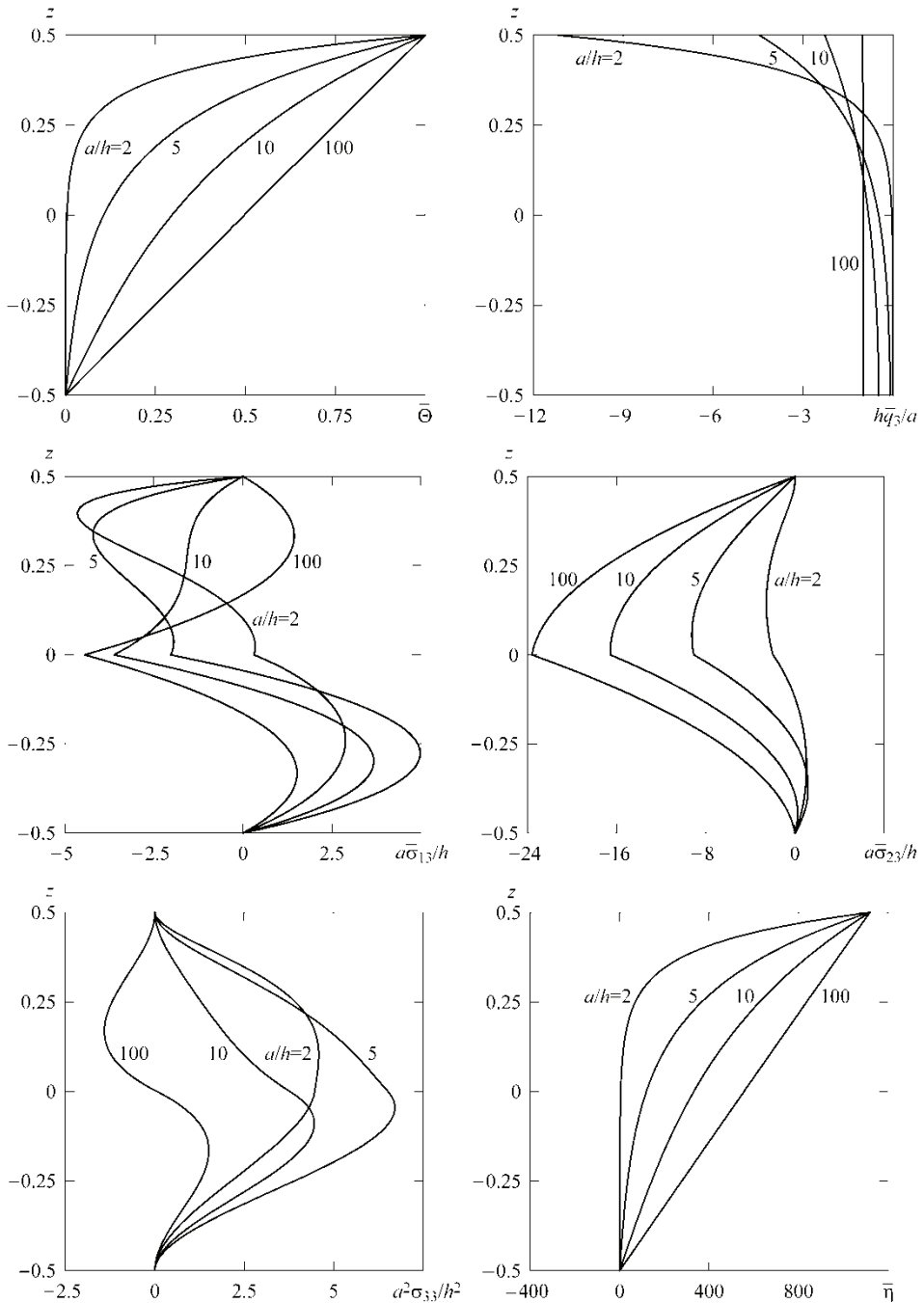


Fig. 2. Distributions of the temperature, heat flux, transverse stresses and entropy through the thickness of the angle-ply plate

The data listed in Table show that the SaS method permits the derivation of exact solutions of plane strain thermoelasticity for thick angle-ply plates $a/h=5$ with a prescribed accuracy using the sufficient number of SaS. Fig. 2 presents the distributions of the temperature, heat flux, transverse stresses and entropy through the thickness of the plate for different values of the slenderness ratio a/h by choosing eleven SaS for each layer. As can be seen, the boundary conditions for the transverse stresses on outer surfaces and the continuity conditions for the heat flux and transverse stresses at a layer interface are satisfied exactly utilizing the constitutive equations (23) and (34).

Conclusions

An efficient method of solving the steady-state problems of 3D thermoelasticity for laminated orthotropic and anisotropic plates has been proposed. It is based on the new method of SaS located at Chebyshev polynomial nodes inside the layers and interfaces as well. The stress analysis of laminated plates is based on the 3D constitutive equations and gives an opportunity to obtain the 3D exact solutions of thermoelasticity for thick and thin cross-ply and angle-ply plates with a prescribed accuracy.

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Трехмерный термоупругий анализ слоистых анизотропных пластин

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Ключевые слова и фразы: метод отсчетных поверхностей; слоистая анизотропная пластина; термоупругость.

Аннотация: Представлен метод отсчетных поверхностей с приложением к анализу трехмерной квазистатической задачи термоупругости для слоистых анизотропных пластин, подверженных температурному нагружению. Показано, что метод отсчетных поверхностей может быть эффективно использован для получения точных решений трехмерной задачи цилиндрического изгиба термоупругих перекрестно армированных композитных пластин с заданной точностью, используя достаточное число отсчетных поверхностей, размещенных в узловых точках полинома Чебышёва.

Dreidimensionale thermoelastische Analyse der blättrigen anisotropen Platten

Zusammenfassung: Der Artikel stellt die Methode der Abzählflächen mit der Anwendung zur Analyse der dreidimensionalen quasistatischen Aufgaben der Thermoelastizität für die anisotropen Platten, die der Temperaturbeaufschlagung unterworfen sind, dar. Es ist gezeigt, dass die Methode der Abzählflächen für das Erhalten der genauen Lösungen der dreidimensionalen Aufgabe der zylindrischen Biegung der thermoelastischen kreuzarmierten Kompositplatten mit der aufgegebenen

Genauigkeit wirksam verwendet sein kann. Dabei wird die ausreichende Zahl der Abzählflächen, die in den Knotenpunkten des Polynoms von Chebyshev aufgestellt sind, verwendet.

Analyse thermoélastique de trois dimensions des plaques anisotropes feuilletées

Résumé: L'article présente la méthode des surfaces de repère avec une annexe pour une analyse du problème quasistatique de trois dimensions de la thermoélasticité pour les plaques anisotropes feuilletées soumises au chargement thermique. Est montré que la méthode des surfaces de repère peut être utilisé avec une efficacité pour l'obtention des solutions précises du problème de trois dimensions de la courbure cylindrique des plaques thermoélastiques armées croisées avec une précision donnée ayant employé un nombre suffisant des surfaces de repère situées dans les points-noeuds du polynome de Tchebichev.

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